An Approach to Astrophysical Time Variable Observation

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1 Abstract

This document accompanies a presentation given for the Rice University Astronomy Seminar and describes a method for subtracting ambient light that can be generalized via a Gram system to measure time variability of optical signals

using a single image exposure. As the RGB Bayer filter projects the wavelength spectrum onto a bandpass basis, the temporal filter herein described projects the temporal function of a signal onto temporal basis using an array of optical modulators. Reconstruction of the original optical signal can then be achieved by solving a Gram system.

2 Analysis

2.1 General framework

For convenience, normalization for the input beam is assumed. For a real system however, a normalizing measurement must also be made so that variations in the input beam do not affect temporal reconstruction.

The time frame is the exposure time, going from 0 to t_0, the length of the exposure.

An input beam of light, $f(t)$, is defined on the same interval as the exposure.

A series of N modulators, $p_n(t)$, is defined on the same interval as the exposure, with modulation possibly varying over time. There are N matching integrating measurement devices, the pixels, whose measurement is defined by m_n.

The beam passes through the modulator medium, thus picking up a modulation. Each pixel then integrates the entirety of the modulated light to measure a single number representing the total measured flux of the beam after undergoing a particular modulation.

$$m_n = \int_0^{t_0} f(t_e - t) p_n(t) dt$$

$f(t)$ is reversed as it is being convolved with the modulation function, so it is evaluated on the modulation frame in reverse.

Intuitively, it can be seen that the modulation imprints temporal information on the input signal which is then integrated at the pixel. The more numerous and orthogonal the modulation measurements for the same input, the more can be said about the input's temporal behavior.

2.2 Case 1: A known input and a single known modulator

This is the normal TetraVue case in which the input beam and modulators are known functions due to our calibration procedure, and they are constants for every single frame.

For clarity, the known variables are:

- $f(t)$, defined only by the laser diode pulse shape and measured during calibration
- $p_n(t)$, defined by the Pockels Cell modulation ramp and measured during calibration

- m_n, defined the pixel's integrated measurement of f(t) modulated by $p_n(t)$

And the camera seeks to find:

- α in $f(t+\alpha)$, the timing delay that characterizes the pulse position relative to the modulation function and by extension, the time of flight and distance

The measurement can then be restated as

$$m_n = \int_0^{t_0} f(t_e - (t+\alpha))p_n(t)dt$$

Assuming a normalized input (which can be made normalized by a measurement without modulation, or indeed any independent modulation), α is the only unknown, and we require only a single modulation function. We can solve for the case that $f(t)$ is a Gaussian and $p_1(t)$ is a half wave sinusoidal (have not done yet).

This should reveal that the single modulation must be monotonic in order to uniquely find α.

2.3 Case 2: A known input with an unknown constant offset and a single known modulator

When the TetraVue camera is taken outdoors and the ambient light overpowers the bandpass filter and exposure limiters, Case 2 is relevant because the input beam is the sum of the known laser diode pulse as well as a linear offset from the ambient illumination. But now, the ambient light complicates $f(t)$ such that

$$f(t) = f_p(t) + f_a(t)$$

where $f_a(t)$ is the portion of the input beam from the ambient light which we do not know.

For clarity, the known variables are:

- $f_p(t)$, the laser diode pulse shape portion of $f(t)$
- $p_n(t)$, defined by Pockels Cell modulation ramp
- m_n, defined the pixel's integrated measurement of f(t) modulated by $p_n(t)$

And the camera seeks to find:

- α in $f_p(t+\alpha)$, the timing delay that characterizes the pulse position relative to the modulation function
- $f_a(t)$, the ambient portion of the input function

A simplification can be made by assuming $f_a(t)$ is a time independent offset because we do not usually expect the solar flux to change appreciably during a short exposure (< 50 us), though it easily varies frame to frame or pixel to pixel. We can then rewrite $f(t)$ as

$$f(t) = f_p(t) + f_a$$

$$m_n = \int_0^{t_0} (f_p(t_e - (t + \alpha)) + f_a) p_n(t) dt$$

$$m_n = \int_0^{t_0} (f_p(t_e - (t + \alpha)) p_n(t) dt + f_a \int_0^{t_0} p_n(t) dt$$

which indicates that the measurement is the sum of the integrated modulated pulse and integrated modulated ambient, but that the ambient has no time dependence, and so the ambient component of the measurement simplifies to the ambient times a known factor. It is a known factor in the TV camera case because we can measure the modulation function of a time independent input signal which is simply the integral of the modulation function. For example, we can measure this integral by pointing the camera towards a bright blank wall and averaging frames that are captured in the same way as normal frames but without pulsed illumination. The exposure length and timing settings must remain unchanged to include the true modulation any time dependent modulation artifacts like ringing.

There are then two unknowns, and two independent equations are needed to solve the system. We assumed a normalized input; therefore, in practice, a third equation and measurement for the no modulation case (or an independent modulated pixel) is needed.

Calling

$$m_1 = \int_0^{t_0} (f_p(t_e - (t + \alpha)) p_1(t) dt + f_a \int_0^{t_0} p_1(t) dt$$

$$m_2 = \int_0^{t_0} (f_p(t_e - (t + \alpha)) p_2(t) dt + f_a \int_0^{t_0} p_2(t) dt$$

$$f_a = \frac{m_2 - \int_0^{t_0} (f_p(t_e - (t + \alpha)) p_2(t) dt}{\int_0^{t_0} p_2(t) dt}$$

$$-m_1 + \int_0^{t_0} (f_p(t_e - (t + \alpha)) p_1(t) dt + \left(m_2 - \int_0^{t_0} (f_p(t_e - (t + \alpha)) p_2(t) dt \right) \left(\frac{\int_0^{t_0} p_1(t) dt}{\int_0^{t_0} p_2(t) dt} \right) = 0$$

Which is an equation with a single unknown, α.

2.4 Case 3: An unknown input and N known modulators

Unrelated to the TetraVue ranging application, but worth ,mentioning for its possible applications in the observation of time variable phenomena within a single exposure using 2d integrating arrays, is the extension to N modulators and an unknown input function.

For clarity, the known variables are:

- $p_n(t)$, defined by a particular modulation scheme
- m_n, defined the pixel's integrated measurement of f(t) modulated by $p_n(t)$

And the camera seeks to find:

- $f(t)$, the unknown input

Motivated by Fourier Series, we consider how to construct the elements of the Fourier Sine Series (FSS) using the integrated measurements to finally obtain a reconstruction of the original input, $f(t)$.

We design the modulators to produce sinusoidal shapes varying between 0 and 1.

$$p_n(t) = \frac{sin(\frac{\pi}{2}nt) + 1}{2}$$

We can then adjust the measurement which is now

$$m_n = \int_0^{t_0} f(t_e - (t + \alpha))\frac{sin(\frac{\pi}{2}nt) + 1}{2}dt$$

to match the FSS coefficients.

$$m'_n = 4(m_n - m_0/2)$$

where m_0 is an unmodulated integration of $f(t)$.

The FSS representation of $f(t)$ is then approximated by

$$FSS(f(t)) = \sum_{n=1}^{N} m'_n p_n(t)$$

and we can discern the dominant temporal features of $f(t)$; specifically, the frequency components up to $\frac{N}{t_0 * 2}$, by the Nyquist limit. This requires being able to modulate the signal on a per pixel basis, perhaps using LCD technology or pixels with different response curves. Furthermore, the super-pixel containing the N separately modulated pixels must receive the same input beam, requiring a potentially large point spread function.

A more versatile formulation is the approximation of the input function as the projection onto N vectors. This constitutes a Gram matrix and a Gram system whose solution supplies the best approximation of the input using the N component functions. Specifically, x represents the the coefficients for each m_n in a linear combination that approximates $f(t)$. The Gram system is

$$Gx = b$$

With G the Gram matrix created by taking the inner product of all the p_n s with each other, x a vector of coefficients which we seek to find, and b composed of m_n.

The Gram approximate of $f(t)$ is

$$GRAM(f(t)) = \sum_{n=1}^{N} x_n p_n$$

We might then optimize the p_n modulation functions to form the most efficient approximation given certain assumptions about the input signal. For example, if we assume the input has a constant slope over the exposure, we can perfectly approximate $f(t)$ with only one modulation function. The more complex we assume the input to be, the more measurements are needed to properly approximate the temporal behavior.

The Gram system, unlike the FSS, allows us to use imperfect modulation functions that are not well described by analytic expressions which is to expected in a realized system. The components of the gram matrix are measured once during a modulator calibration by taking multiple frames of a short pulse of light across the exposure.

3 Astrophysical Time Series Observation

The concept is to measure the light from the same star with a group of pixels, each having a different modulation. In the same way that an RGB array acts as spectral separator, a modulation array acts as a separator of temporal information that occurs during the integration period. Usually, if temporal resolution is desired, one simply takes more exposures with shorter integration periods. This technique allows the user to find temporal behavior during the exposure itself.

3.1 Point Spread Function Variation During Exposure

The limited angular resolution and atmosphere spread point source light over a distribution of pixels, which are then summed to obtain a single photometric count. The atmospheric fluctuations are on millisecond timescales and should not be measurable by the proposed system.

Telescope stabilization and tracking must be precise enough not to move the star's center within the PSF during the exposure, because this would register as a confounding time variability. The tracking must be precise to less than a few pixel.

An optical blurring filter can be used to achieve particular PSF.

3.2 Potential Mechanisms

Different modulators may be useful depending on the time resolution, pixel pitch, and wavelengths required. Below are illustrated a few of the modulators thus far considered.

3.2.1 Liquid Crystal Display

The ideal solution would be a programmable modulation for each pixel. With current state of the art LCD technology, it is possible to place an individually programmable modulator in front of each pixel for pixel pitches as small as 27.5 microns. This means the modulation function and the number of modulators can be chosen on the fly or perhaps in response to previous measurements. Because they are used for RGB displays and have undergone decades of industry research, LCDs are optically broadband and highly robust over varied operating conditions. Perhaps the only drawback of the LCD is a refresh rate of at most 120 Hz.

An extra benefit of an LCD placed located at the focal plane is its dual use as locally adaptive shutter to enhance the Dynamic range of the sensor. Dynamic range problems have recently arisen by which increasingly larger telescopes are saturating once dim reference star. Saturation of stars can lead to banding and global offsets for the entire chip leading to erroneous errors that are difficult to correct. It is then desired to attenuate the brightest objects without attenuating the dim objects. The LCD can be used to variably attenuate all stars to achieve non saturating measurements. In practice, a test image would be produced with no attenuation that identifies the brightest objects. The effective exposure over the brightest objects in the array can then be adjusted by changing the LCD only over those objects.

3.2.2 Micro Patterned Analyzers

If pixel pitch becomes a limiting facts for an LCD approach, a polarizer grid can be placed directly on the silicon sensor, either as part of the 1st metal layer during sensor fabrication, or as an added ion beam etched wire grid polarizer. The superpixel would be composed of 4 states: 0, 45, 90, and 135 allowing for a complete identification of Stokes's parameters. There are several manufacturers for pixellated or micro patterned polarizer grids that have achieved optical wire grid polarizers as small as 5.6 microns. On top of the grid, a global polarization rotating mechanism rotates the polarized input beam through 180 degrees. When analyzed by the 4 polarizer states, this yields 4 distinct modulation functions. The polarization can be rotated globally by a a single ITO coated liquid crystal, a mechanically rotating half-waveplate, or a Pockels Cell.

It should be noted, that there is limited broadband use for half waveplate or Pockels Cell, but compensating calibrations may prove useful and megahertz modulators like a Pockels Cell may eventually prove useful.

3.2.3 Chip modulation

An on the rise practice in High Dynamic Range sensors is to use dual simultaneous exposures of different lengths to effectively increase the dynamic range of the sensor. Typically, every even rows are exposed at one wavelength, and odd at another. This is done by using multiple transfer gates and sense nodes. A similar approach can be taken to incorporate different modulations for N rows in the final read out.

3.3 Viability

We compare this multiple modulator scheme to the standard multiple exposure scheme. In particular, we look at noise properties and achievable temporal resolution.

3.4 Noise Considerations

3.4.1 Quantum Nature of the Measurement

A polarizer is subject to probabilistic quantum effects which are here considered. Monte Carlo simulations suggest that the noise contributions due to quantum uncertainties are dwarfed by shot noise.

3.4.2 Signal Loss

Depending on the modulation scheme, varying amounts of signal will be lost. The LCD or patterned polarizer schemes require an input polarizer which makes for a 50 percent loss assuming unpolarized stellar light. The LCD and optical modulators have further wavelength dependent transmission losses ranging from 10 to 40 percent.

Only the chip modulation method would not affect signal through optical losses. But all schemes have an additional 50 percent loss relative to a normal exposure because the average modulation has 50 percent transmission.

Another increase in noise relative to a conventional setup is the reduction in pixels that can be used to compute total flux. Typically, all the signal in the PSF is summed to measure the magnitude. However, in our scheme, there is a N^{-1} reduction in pixels to sum for each modulation. The total flux, which is the sum of all modulations, is reduced by a factor two for equivalent exposures. But the flux for each modulation, is reduced by a factor of N^{-1} which will leads to higher noise relative to a conventional setup.

Therefore, to achieve the same noise properties for magnitude measurement as a conventional setup (no modulators), exposures would have to be increased 2 to 6 times. The temporal behavior however, has been shown to be robust all the way to S/N levels of 20 to 1.

3.5 Pipeline Considerations

Compared to a typical data pipeline, a few additional steps are required to extract temporal data. We review here some of the complications.

3.5.1 Interpolation

Assuming a repeating grid like modulation scheme, interpolation can be done to create N aligned images at the original sensor resolution. However the interpolation is more complicated than typical RGB interpolation methods like bilinear with ratio or difference constancy. This scheme requires interpolating according to the PSF. To illustrate, if we are interpolating across 2 pixels that straddle the sides of the peak of a PSF, the interpolated pixel must ride the peak. This can be done by PSF fitting all modulated images separately, averaging the resulting PSFs, and finally weighing interpolated pixels according to the normalized PSF.

3.5.2 Global Variation

Global or systematic temporal variability will be present in the system through such effects as telescope wobble, atmospheric changes, light contamination, or geometric translation due to atmospheric viewing angle. All sources in a given ccd will experience the same systematic variation during the exposure, which forms the basis of a reference modulation. Any temporal change in the entire field, or subsections of the field, can be attributed to global temporal variability, and this component of variability can then be subtracted from each object's temporal behavior.

4 Conclusions and Future Work

I solved a problem that lets a time of flight camera go outside and found a way to generalize it using a Gram system. But is it actually useful? Astronomers do not even consider Bayer filters because they cannot be changed, sacrifice spatial resolution and increase noise. They would rather just take multiple frames in different filters. Indeed, all but the in chip modulation techniques require a sacrifice in total transmission on the order of 2-4 times less measured photons.

One possible advantage is the ability to grab all the information in a single frame and so lower the data storage requirements by a factor of N.

A possible advantage is the ability to do large field of view surveys to look for transients. While the system can only reconstruct the time series function down to frequencies of N/(2N T) by the Nyquist Frequency (T the exposure, N the number of modulators), it is very easy to detect whether the object in question is behaving like a very ordinary constant in time object.